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A critical discussion of cognitive development theory and information processing theory to assess Number Sense in typical developed first graders.

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Abstract

The development of a number sense and therefore, of mathematical cognition in first graders will determine critical aspect of future life. However, the pioneering concepts of cognitive and mathematical development that have influenced current interventions in the classroom may ultimately undermine children's innate ability to perceive mathematically and, therefore, intervene in a retrospective manner. In the present study, by reviewing the literature, the theory of cognitive development and the theory of information processing unfolded through a critical discussion in relation to their ability to interpret the number sense ability of first graders, but also in their ability to enhance it using manipulatives and by evaluating working memory respectively. From the analysis of these interventions, it appears that manipulatives seem to involve shallower cognitive paths of the first graders, while the evaluation of the working memory leads to slightly better results of the development of number sense. Alternative ways of developing a sense of numbers are discussed.

Keywords

Number sense; First Graders; Theory of Cognitive Development; Information Processing Theory

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Introduction

Theory of cognitive development [1] and Information Processing Theory [IPT, 2], pioneer concepts in cognitive and mathematical development, attempted to frame the development of Number Sense (NS) in typical developed (TD) first graders (FG). The use of mathematical manipulatives and Working Memory (WM) training, respectively for each approach, are widespread for tackling counterproductive factors blocking NS. However, both theories created divided opinions regarding the onset of NS, a phenomenon which appears above poor results produced when such approaches, especially manipulatives, are utilized. The meta-analysis of Peltier et al. [3] which included studies regarding the usage of manipulatives to expand NS in all elementary stages -addition and subtraction skills for FG-, exposed a significant point based on small effect sizes regarding its non-effectiveness contrast to typical training in TD FG, an age group who seem to be devaluated by Piaget's [1] theory. Between their nine settled variables, their findings revealed that manipulatives were effective in enhancing NS of children who were recognized as having a disability or who were at risk of being identified as having one whereas results produced only when one-digit performances was given. Thus, manipulatives may favor specific cognitive -or emotional- difficulties instead of NS. On the other hand, the meta-analysis of Khatib et al. [4] grounding on moderate effect sizes, they incarnated the fact that NS of TD FG may benefit more when selectively triggering WM's operation. By utilizing digital high-control visuo-motor and enjoyable stimuli, TD FG been motivated to produce good standards of two-digits mathematical performances. This reveals the cognitive power of young children that seeks the influence of deeper cognitive pathways by complex tasks. Yet, due to difficulties in far-transference of skills for numbers between 15-20, the question if WM and thus NM can indeed be assessed and reinforced within the classroom, emerges [5]. To this point, NS in FG will by critically defined, for the conceptualization of the effectiveness of the mentioned strategies, by analyzing relevant metanalyses and trials, to arise.

Making sense the Number Sense of FG

According to National Council of Teachers of Mathematics [NCTM, 2000] the age of six (FG) is a period of crucial focus for NS, as this will determine mathematical development and thus, brain development and quality of future life. The transition from kindergarten to FG can work as a stress device which will negatively influence the emotional domain of schoolchildren and thus, math achievement and when enrolled, they already have significant individual disparities in their numerical performance, lurking on socio-economic-status (SES) differences [6]. Yet, even though a wider prevalence rate that indicates 6% of FG with impaired NS [7] most elementary schools promote mass education within language-grammar orientation [8]. Thus, these impairing factors of NS should be on the scope when interventions designed. NS, concerning FG, is the key component for enabling them to achieve fluency in mathematic major concepts: addition and subtraction using numbers mainly until ten, and twenty [9] and despite its vitality, recent data are characterized by a confusion regarding its meaning [10]. However, to facilitate its understanding for discussing reinforcement strategies, the present study will use one of its first and holistic definitions: NS, as a component of numerical cognition, indicates the cognitive power of children to perceive and mentally represent the essence of a number and/or numeral fact and by manipulating it, to transfer it in the real world [11]. Yet, as mentioned, the data produced by the Piagetian theory and IPT for the onset of NS can characterized by turbidity.

Gradual VS universal nature

Even though Piaget [12] believed that babies are born with a blank cognitive slate in which eventually build mathematical knowledge by gathering numerical information -such as time- from their surroundings, he stated that children do not have NS until they were about five-to-six years old, and that they couldn't comprehend the concept of number conservation until then. Similarly, conceptual understanding of mathematics until they're six or seven years old is absent [12]. Even though preschoolers find themselves within the preoperational stage (age of 2-7) with centration, conservation, symbolic representation but egocentrism to present Piaget's [13] argument for low-NS-standards between ages of 5-7 can be dismissive for the effectiveness of current NS manipulatives interventions in kindergarten preparing them for the FG. Especially when it comes to abilities such as finger-counting, identifying, and formatting numbers, abilities which are constructed upon linguistic skills [14]. However, IPT opposite to Piaget who he defined NS as a loot resulted by a specific developmental trajectory, assumes firstly that NS exists from infancy and secondly, that it has a continuum development, which grounds on the approximate number system (ANS), a cognitive framework that aids in the assessment of magnitudes instead of utilizing language device or symbols [15]. IPT considers for an operational correlation between NS and WM, as fMRI studies have indicated a neuronal link on them when numbers were present as the center of attention, manipulated, and stored by schoolchildren [16], giving to WM a vital role in mathematical cognition. WM is a memory system which holds a unique role within the data encoding, storage, and retrieval stages of memory with controversial capacity issues [17]. As far as WM neurologically identifies with NS, unfolding the Multicomponent-Model of WM [18], NS can be conceptualized as a multimodal entity which directs attentional control to perceive either visual or verbal mathematical information and mentally manipulate them by utilizing the unlimited attentional central executive, the visuospatial sketch-pad, the phonologic loop, and the limited episodic buffer which can manipulate and store around 7±2 chunks of information, respectively. However, the Embedded Process Model turned the problem of capacity limitation in the attentional span in-

stead of storage, bringing in the discussion to either internal or external distractors [17]. This model which perceives WM as a part of Long-Term Memory (LTM), considers that its operation contributes to connect past and present mathematical knowledge and not controlling significant cognitive processes. Regarding distractors reducing the attentional span, holding the aspect of Cognitive Load Theory [19] which considers for minimizing the reductive relationship of extraneous loads and WM, the whole classroom or teaching techniques may also be considered as factors that reduce NS. However, for both explanatory theories, the NS will be characterized by capacity deficits either in storage or in attention.

As mentioned, the development of NS may be degreased by external factors such school-curriculum, transition's anxiety, and SES background [6; 8]. However, Piaget [12; 13] attributed the mathematic difficulties in the absence and/or immature NS at the age of six, whereas WM theories assumes for distractors that may impair NS components [19]. Regardless the identification for NS's inherent nature, ITP considers for NS capacity undefined issues [18; 17], whereas Piaget's theory is biased towards the advantageous position of the FG, if their NS is universal. Factors of cognitive immaturity are highlighted yet, self-trust and motivation [20; 21] will be not promoted as two key-factors in the emotional domain of FG, grounding the mis development of NS, promoting the presence of dyscalculia [22]. This phenomenon which captures 3-7% of schoolchildren finishing elementary education [22] is a term refer to problems in processing numerical information, perform one-digit addition and subtraction, memorizing arithmetic facts and calculating, reflected by impairments in NS [23]. Piaget's disadvantageous belief of absent/immature NS in FG is reflected by the counterproductive use of manipulatives. WM training on the other hand, seems to have a role on enhancing NS.

Manipulatives and Number Sense

Piaget suggested teaching methods of math to FG which are opposed to the cognitive barrier that he perse have attributed to them, as research confirms. By degrading the superiority that visual representations have on empowering the mental imagery span of children Guarnera et al. [24] suggested concrete things such as symbols to exploit symbolic thought to advantage the expansion of NS and satisfy the co-development of memory and imagination [13] with problem solving tasks such as blocks. Regarding the use of manipulatives, Woods et al. [25] showed that the usage of concrete representations of numbers accompanied by verbal instructions benefits FG to grow their NS. Lantz and Caitlyn [26] agreed that manipulatives, e.g., base-ten blocks, did assist FG to exhibited addition and subtraction computation performance that was significantly higher than that of third graders who had received traditional teaching. However, they highlighted two limitations that may agree with one cognitive barrier captured by Piaget. Opposite to symbolic thought, egocentrism, the inability to understand that another person's view may differs from their own, may lead personal maladaptive numerical beliefs to be reinforced. To this axis, FG learned to use manipulatives without learning the underlying mathematical framework. They failed in applying problem-solving skills of counting, adding, and subtracting without the use of manipulatives and constantly reminders from teachers [26]. Therefore, manipulatives may reinforce the presence of cognitive egocentrism and since manipulatives cannot always connect concrete and abstract mathematical concepts, block on the development of NS may arise, by creating stress, and a restrictive perception that mathematics is difficult to acquire [27]. This Piagetian strategy, depended on constructivist mathematical learning by targeting assimilation and accommodation processes [13], engages FG by exploiting their cognitive pathways in a shallow way, non-exploiting deeper levels which will boost the episodic capture of knowledge and may favor the reinforcement of this abstract ability [28].

Working Memory-training and Number Sense

To this mathematic conceptual gap, De Vita et al. [29] holding a IPT perspective, they emphasized in the enhancement of WM domains to reinforce FG' NS, using mainly digital visual representations in contrast to Piagetian argument to use concrete objects at first. Their assessment yielded strong performance of FG in numerical-verbal low and high-control WM skills and Visuo-Spatial high-control WM skills reflecting good standards of NS. This means that FG are benefit when they are using a combination of tactics, principally a verbal approach augmented with visual-spatial representations [30]. Numerical-verbal low and high-control WM skills assessed using digit forward and backward recall tasks of 0-9 respectively, showing counting ability and number conceptualization. Visuo-spatial high-control WM skill was tested using the visuo-spatial dual task by watching a path taken by a small frog on a 4x4 matrix with a red square and the children had to recall the frog's starting position on each path and touch on the table when the frog reached the red area, indicating good quantitative and spatial understanding. This excellence may be explained in the capture of attention [31], due to the concept of the exercises and in the increase of motivation to participate [32], whereby risk-taking behaviors, such as recall mathematical concepts and button press, reinforced the mnemonic representations of WM [33]. As a result, it appears that the ability to remember and manipulate numerical information while performing complex mathematical tasks is important not only in primary school and in children with difficulties, but also in TD FG when formal education begins. These findings also indicate the gradual maturity of high-control WM processes in tandem with the simultaneous maturation of the prefrontal cortex between the ages of 5 and 7 [29]. However, when numbers between 15-20 were about to be added or subtracted [34; 35), poor far-transference of WM skills to real mathematical concepts was emerged. Hence, if attention of FG can proceed such multifactorial tasks but cannot long-term storage such skills, then the storage capacity should be targeted [29]. Despite the non-far transference of skills in 15-20

tasks, the appearance of the ability to remember, manipulate and transfer complex mathematical concepts and two-digits numbers, reflects the superiority of this IPT intervention to the Piagetian one, promoting a slight better advantage in NS.

Discussion

The present study targeted to unfold Piaget's theory and ITP with regards to critically explain NS in FG, to produce data for the effectiveness of their two-representative umbrella-interventions. Initially, this literature research revealed that there is not a single randomized clinical trial examining the present concern. This study concluded that Piaget's theory of cognitive development seems to be biased in relation to the development of NS in the age of 6. On the other hand, ITP seems to be stronger, both in defining and developing this abstract construction. WM-training led FG to manipulate two-digit numbers in contrast to manipulatives that produced outcomes only when it comes to one-digit performances on children with specific cognitive and/or emotional difficulties. From the analysis of the effectiveness of manipulatives and WM-training on NS it arises that those Piagetian beliefs may have underestimated the intelligence of FG and the vitality of exploiting children' visual systems when teaching symbolic concepts. The non-effectiveness of utilizing concrete things, where manipulatives seems that they cannot influence children' mental imagery, may promote the presence of math anxiety. WM training firstly, assisted individuals to perform complex mathematical tasks and secondly, favored the need of assessing abilities yet, may not reflect far-transferable abilities in NS due to WM immaturity, whereby both intelligence and WM immaturity may ground on SES differences, a factor that have ignored from both theories. Hence, to set individual intelligence as of high priority, to promote self-trust and motivation and to overpass SES differences and language-grammar orientation, the discussion should turn to more holistic interventions.

Tackling counterproductive factors

Regarding individual intelligence and SES, whereby intelligence indicates the way that an individual learns and perceives outer environment, the application of the Multiple Intelligence Theory [MIT, 36] may enlighten the development of NS in TD FG. The metanalysis of Ferrero et al. [37] on MIT, despite the methodological and practical issues produced by MIT applications, revealed that FG achieved fluency in mathematics, as much as in literacy. Assessing the superiority of individuals between the eight different levels of intelligence as defined by Gardner [36] can indicate to teachers which learning technique is best for each individual child. For example, if a child has high levels of visual-spatial intelligence, then WM-training with visual-spatial stimuli is an ideal method. Respectively, high levels on bodily-kinesthetic intelligence may indicate the need to use manipulatives for enhancing NS. Regarding SES differences and language-grammar orientation, Kumon method should be discussed. The metanalysis of Orcos et al. [38] yielded that the earlier students begin learning mathematics with the Kumon method, the more likely they are to achieve a level of knowledge above their school level, demonstrating the method's potential in the teaching and learning of mathematics at the early childhood and primary education levels. This approach isn't meant to be used to grade students' NS; rather, it's meant to be used to assess, exploit, and build personalized study strategies. Kumon instructors intend for pupils to study at a level of difficulty that is challenging enough to keep them engaged but not too difficult to discourage them as they progress. Students are encouraged to study at home for 15–30 minutes five days a week, allowing parents to interact with their children's NS. However, Kumon method is utilized in extra-curriculum centers and may favor high SES families.

Limitations

As mentioned, the Kumon method and MIT may produce good results in the development of FG NS, but they should be affordable to various family SES and address methodological issues, respectively. With regards to manipulatives and WM training first, no metanalysis or RCTs found in global libraries to address the combination or the contrast of such utilization. Therefore, the conclusions of the superiority of ITP in better assessing NS in contrast to the Piagetian theory of this present critical discussion may be biased towards the interpretation of the separated results from each metanalysis and trials. Hence, this study suggests for more clinical trials to cover the specific literature gap.

Conclusion

Information processing theory advantages first graders in contrast to Piaget with regards to their ability in sensing numbers and numerical facts. Information processing theory applications produced better transference of the intervention in real life that Piagetian interventions regarding the development of number sense.

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